

Athena Visual Studio Partial Differential Model Tutorial

- ❖ Start **Athena Visual Studio**
- ❖ From the **File** menu select **New**.
- ❖ You are in the **Process Modeling** tab.
- ❖ Click **Modeling with PDEs with Diagonal E Matrix**.
- ❖ Select **A Blank Document** and click **OK**.
- ❖ Enter your model data, initial conditions and equations, and the Athena solver data and options as described in this tutorial.

When you are done:

- ❖ From the **File** menu click **Save**.
- ❖ Navigate to the folder where you wish to save and enter a proper filename for your model.
- ❖ From the **Build** menu click **Compile**.
- ❖ From the **Build** menu click **Build EXE**.
- ❖ From the **Build** menu click **Execute**.

Initial-Boundary-Value problems arise in the analysis, design and control of processes that are modeled in two dimensions, one of which can be traversed by forward integration. In this section we consider only two-dimensional systems; however, the approach described is also applicable in higher-dimensional problems. Examples can be found in chemical reactor engineering, various combustion processes, atmospheric chemistry, industrial design and various biological systems. The complexity of realistic models makes it very

difficult to determine the effects that small changes in their physical and chemical parameters would have on the predicted output of the process. Sensitivity analysis of such systems can reveal an abundance of information about the underlying mechanistic steps and provide information for model development, optimal experimental design and parameter estimation. Partial Differential Equations models take the form:

$$\mathbf{E} \frac{\partial \mathbf{u}}{\partial t} = \mathbf{F}(t, x, \mathbf{u}_x, \mathbf{u}_{xx}; \boldsymbol{\theta}) \quad \text{where} \quad \mathbf{u}_{xx} = \frac{1}{x^m} \frac{d}{dx} \left(x^m \frac{d\mathbf{u}}{dx} \right) \quad \text{and} \quad \alpha < x < \beta$$

Initial Conditions Boundary Conditions

$$\mathbf{u}(t_0, x) = \mathbf{u}_0(x) \quad \mathbf{E} \frac{\partial \mathbf{u}}{\partial t} = \mathbf{F}(t, \alpha, \mathbf{u}_x; \boldsymbol{\theta}) \quad \mathbf{E} \frac{\partial \mathbf{u}}{\partial t} = \mathbf{F}(t, \beta, \mathbf{u}_x; \boldsymbol{\theta})$$

where $\mathbf{u}(\)$ is a state vector of unknowns (usually temperature, pressure and composition), $\boldsymbol{\theta}$ is a vector of known parameters pertinent to the process we are modeling, $\mathbf{u}_x(\)$ represents the first order derivative of the state vector $\mathbf{u}(\)$ with respect to the dimension x , and $\mathbf{u}_{xx}(\)$ represents the second order derivative of the state vector $\mathbf{u}(\)$ with respect to x (in the appropriate coordinate system). Initial-Boundary value models are ordinarily used to model unsteady-state reaction and diffusion as well as steady-state fixed bed reactors with significant gradients in the radial direction. They can be solved using **PDAPLUS**, that combines a powerful modified Newton algorithm with a fixed leading coefficient backward difference formula for the approximation of the first order time derivative and various discretization schemes (such as Finite Differences, Global Orthogonal Collocation and Collocation on Finite Elements) for the spatial derivatives.

Tutorial: Diffusion in Nylon-12 Food Packaging

This example tutorial has been created to test the functionality of Athena Visual Studio in dealing with the solution of partial differential equations. An additional feature, the simultaneous solution of an ordinary differential equation, which is valid only on the right boundary is also demonstrated.

The transport of a migrant from polymer to food is described by the following set of partial and ordinary differential equations:

$$\frac{\partial C_P}{\partial t} = D_P \frac{\partial^2 C_P}{\partial x^2}$$

$$\frac{\partial C_F}{\partial t} = -D_P \alpha_v \frac{\partial C_P}{\partial x} \Big|_{x=L_P} \quad \alpha_v = \frac{A}{V_F}$$

$$t = 0 \quad C_P = \rho C_{P_0} \quad C_F = 0$$

$$x = 0 \quad \frac{\partial C_P}{\partial x} = 0$$

$$x = L_P \quad C_P = K_{PF} C_F$$

We wish to solve for the concentration of the migrant in the food C_F as function of time. Also we wish to plot the migrant concentration in the polymer C_P as a function of time at different spatial locations.

The values and description of the parameters for this process are given in the table below:

Model Parameters and Physical Properties	Description and Units
$D_P = 3.0 \times 10^{-14} \frac{\text{m}^2}{\text{s}}$	Diffusion coefficient in polymer
$K_{PF} = 1.4$	Partitioning coefficient
$L_P = 10^{-4} \text{ m}$	Polymer thickness
$V_F = 10^{-4} \text{ m}^3$	Solution volume
$A = 2.0 \times 10^{-2} \text{ m}^2$	Contact area
$\rho_P = 1000.0 \frac{\text{kg}}{\text{m}^3}$	Polymer density
$C_{P_0} = 2300.0 \frac{\text{mg}}{\text{kg}}$	Migrant initial concentration

Let's define the following variables:

$$\xi = \frac{x}{L_P} \quad D_V = \frac{D_P}{L_P^2} \quad D_S = \frac{D_P A}{L_P V_F} \quad u_1 = \frac{C_P}{\rho C_{P0}} \quad u_2 = \frac{C_F}{\rho C_{P0}}$$

Then the model equations can be written as:

$$\begin{aligned} \frac{\partial u_1}{\partial t} &= D_V \frac{\partial^2 u_1}{\partial \xi^2} & \frac{\partial u_2}{\partial t} &= -D_S \frac{\partial u_1}{\partial \xi} \Big|_{\xi=1} \\ t=0 & \quad u_1 = 1 & \quad u_2 &= 0 \\ \xi=0 & \quad \frac{\partial u_1}{\partial \xi} = 0 \\ \xi=1 & \quad u_1 = K_{PF} u_2 \end{aligned}$$

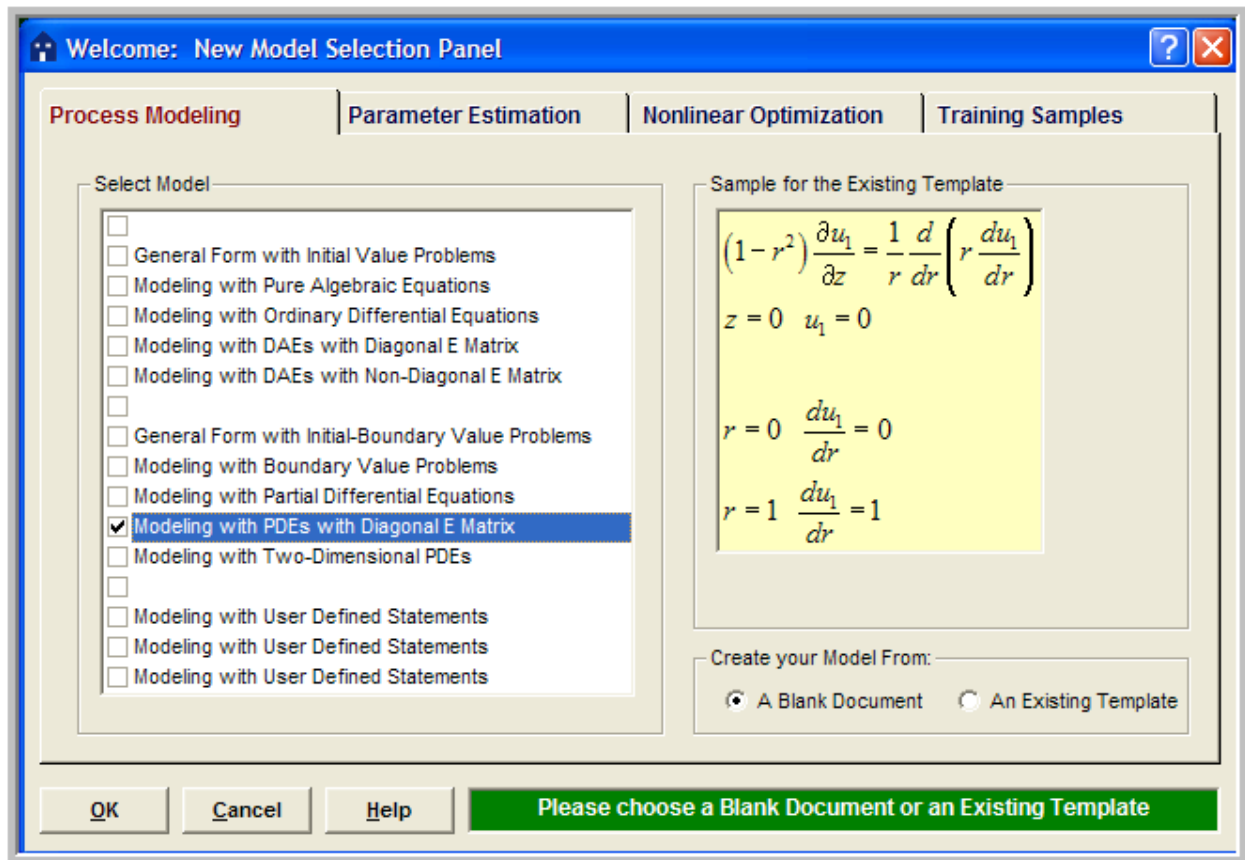
This example tutorial is already precoded in Athena Visual Studio. If you do not wish to type the code on your own you may access it by doing the following:

- ❖ Open **Athena Visual Studio**
- ❖ From the **File** menu click **New**
- ❖ Select the **Training Samples** tab
- ❖ Select the **Dynamic Diffusion with Equilibrium Adsorption** sample
- ❖ Click **OK**

Implementation in Athena Visual Studio

The following step by step process describes the model implementation in Athena Visual Studio

- ❖ Open **Athena Visual Studio**.
- ❖ From the **File** menu, choose **New**.
- ❖ The Welcome: New *Model Selection Panel* window appears.



- ❖ Select the *Process Modeling* tab
- ❖ Select the *Modeling with PDEs with Diagonal E Matrix* option.
- ❖ Choose *A Blank Document* and click **OK**.

Type your source code in the new window. The source code contains standard modeling sections (see description in the next sections below); it may also contain calls to the available math and engineering procedures as well as user-defined procedures.

Writing the Source Code

You must enter a minimum of four sections in order to create the partial differential equations model with diagonal $E()$ matrix. The first section labeled **@Initial Conditions** is used to insert initial values for the state variables vector. The second section labeled **@Model Equations** is used to enter the model equations. The third section labeled **@Boundary Conditions** is used to enter the boundary conditions. The fourth section labeled **@Coefficient Matrix** is used to enter the diagonal elements of the $E()$ matrix. A data section not labeled by Athena Visual Studio may also be used to enter all the data pertinent to the model. The data section may also contain declaration statements for all model variables, parameters and constants. This section, if used, must be the first one in the model. The declaration of the model variables, parameters and constants must be done in accordance the Athena Visual Studio syntax rules shown below:

Data Section

In the data section (which may not be needed and is not labeled by Athena Visual Studio) the user simply enters the problem data and various constants as shown below. In this example the user enters values for the diffusion coefficient in the polymer, the thickness of the polymer, the contact area and the solvent volume, as well as the partition coefficient and the initial concentration and polymer density.. The Athena interpreter treats any line that begins with the exclamation mark **!** as a comment. It is mandatory and strongly recommended that the users declare all the problem variables, parameters and constants. All variables in Athena are either real double or single precision or integer long. Character and Logical variables are also allowed. The following source code may be entered for this example (If you use the Windows Copy and Paste commands to enter this code into your Athena Visual Studio project, please beware that invisible format symbols may also be copied and cause the compilation of your code to fail):

```
! Declarations and Model Constants
!-----
Global Kpf, Dp, Vf, Ac, Rhop, Cpo, Lp As Real
Global Dv,Ds As Real

Kpf=1.4                ! Partition coefficient
Dp=3.0D-14            ! Diffusion coefficient in polymer, m2/s
Lp=1.0D-4             ! Polymer film thickness, m
Ac=2E-2               ! Total contact area, m2
Vf=100E-6             ! Volume solvent, m3
Rhop=1000.0           ! Density of polymer, kg/m3
Cpo=2300.0            ! Initial concentration in mg/kg

Dv=Dp/Lp^2*3600.0     ! 1/hr
Ds=Dv*Ac*Lp/Vf       ! 1/hr
```

Declaration of Variables

Global Variables: To declare global variables in the Athena Visual Studio environment you must use the **Global** keyword as the examples below illustrate:

```
Global x, y, z, krate As Real  
Global Skount, Ncc As Integer  
Global myName As Character  
Global myDecision As Logical
```

In the above statements the variables *x, y, z, krate* will be treated as double precision and will be accessible by all modeling sections. Similarly the variables *Skount, Ncc* will be treated as integer and be accessible by all modeling sections. Character variables are assigned as Character*132 from the Athena Visual Studio parser. Single precision variables cannot be declared **Global**.

Vectors and matrices can be declared in a similar manner. The array size, type and number of dimensions are declared with the **Global** statement. The elements of the array can be referenced by an integer index number, which runs from one (or zero) to the maximum number declared in the **Global** statement:

```
Global y(10), c(0:5), a(4,50), b(2,4,6) As Real  
Global istate(5) As Integer
```

Local Variables: To declare local variables in the Athena Visual Studio environment you must use the **Dim** keyword as the examples below illustrate:

```
Dim Temp, Pres As Real  
Dim TotalFlow As Single  
Dim i As Integer
```

In the above statements the variables *Temp, Pres* will be treated as double precision, where as the variable *TotalFlow* will be treated as single precision; these variables will be accessible only at the section where they have been declared. Similarly the variable *i* will be treated as integer and will be accessible only by the corresponding modeling section where it has been declared.

Vectors and matrices can be declared in a similar manner. The array size, type and number of dimensions are declared with the **Dim** statement. The elements of the array are referenced by an integer index, which runs from one(or zero) to the number declared in the **Dim** statement:

```
Dim c(10), p(4,50) As Real  
Dim streamEnthalpy(10) As Single  
Dim irow(5) As Integer
```

Parameter Statement: Use the **Parameter** keyword to define named constants as the examples below illustrate:

```
Parameter y=2.0, z=4.0 As Real  
Parameter Skount=1, Ncc=4 As Integer
```

In the above statements the variables y , z will be treated as double precision and their numerical values will be accessible by all parts of the modeling code. Similarly the variables $Skount$, Ncc will be treated as integer and their numerical values will be accessible through out all the modeling sections. The **Parameter** keyword is only allowed in the data section of the Athena Visual Studio modeling code. If it is used in the other modeling sections it will be ignored. You may view the generated Fortran code to see how the parser interprets the **Parameter** keyword.

Important Note: Always remember to declare all of your variables. Athena treats **Real** variables as double precision, **Integer** variables as 4-byte integers, **Character** variables as Character*132 and **Logical** variables as **.True.** or **.False.** Single precision variables are only allowed if are declared as local with the **Dim** keyword.

Fortran 95 Declaration Statements: You can insert Fortran 95 declaration statements by prefixing them with the double dollar sign. Below please see a list of Fortran 95 declaration statements that you can insert in your Athena code. Consult your Fortran 95 manual for the syntax rules of variable and constant declarations:

```
$$Integer, Parameter:: dp=Kind(1.0D0)  
$$Integer, Parameter:: sp=Kind(1.0)  
$$Real(Kind=dp):: v1,v2  
$$Real(Kind=sp), Dimension(3):: a1,a2  
$$Integer:: I1, I2  
$$Character(Len=3):: s2,s3  
$$Character(Len=10), Dimension(2):: s1  
$$Logical:: Done  
$$Real(Kind=dp), Dimension(:), Allocatable:: w
```

We are now going to describe in detail the various steps involved in writing the differential model for this example in the Athena Visual Studio environment. The modeling code is NOT case sensitive.

Initial Conditions

In the Initial Conditions section the user must enter the initial values for the unknown state vector. The initial values are required by the algorithm in PDAPLUS to start the integration. The user must do the selection of the unknown state variables. The user must also make sure that he/she has a well-defined system where the number of equations is equal to the number of unknowns. The unknown state vector is represented by the variable $U()$ in Athena. For our example we choose $U(1)$ to represent the dimensionless concentration in the polymer and $U(2)$ to represent the dimensionless concentration in the food. To enter the heading for the Initial Conditions section for our example:

- ❖ From the *Model* menu choose *Initial Conditions* (or **Hit F11**)
- ❖ Enter the source code as shown below for our example.

```
@Initial Conditions
U(1)=1.0           ! Dimensionless concentration in polymer
U(2)=0.0           ! Dimensionless initial concentration in solvent
```

Model Equations

In the Model Equations section the user must enter the functions that describe the physical process. For example these functions may simply indicate the rate of change of the concentration of miscellaneous chemical components. The vector $F()$ is reserved in the Athena environment to represent the values of these functions. For our example $F(1)$ is used to represent the material balance equation for the concentration of the migrant in the polymer, while $F(2)$ is used to represent the material balance equation for the concentration of the migrant in the solvent. To enter the Model Equations section for our example

- ❖ From the *Model* menu choose *Model Equations* (or **Hit F11**)
- ❖ Enter the source code as shown below for our example.

```
@Model Equations
F(1) = Dv * Uxx(1)
F(2) = 0.0
```


Boundary Conditions

In the Boundary Conditions section the user must enter the functions that describe the physical process at the boundaries of the space domain. For example these functions may simply represent flux conditions, state vector values or a mixture of both. The vector $\mathbf{F}()$ is reserved in the Athena environment to represent the values of these functions. The vector $\mathbf{U}_x()$ is used here to represent the first order spatial derivative and the symbol X is used to indicate the value of the space variable on the boundary. The variable **LEFT** is reserved in Athena to indicate the left boundary location, and the variable **RIGHT** is reserved to indicate the right boundary location. To enter the heading for the Boundary Conditions Section:

- ❖ From the *Model* menu choose *Boundary Conditions* (or **Hit F11**)
- ❖ Enter the source code as shown below for our example.

```
@Boundary Conditions
If (LEFT) Then
  F(1) = Ux(1)
  F(2) = 0.0
Else
  F(1) = U(1) - Kpf * U(2)
  F(2) = -Ds * Ux(1)
EndIf
```

Coefficient Matrix

Finally in the Coefficient Matrix section the user must enter the diagonal elements of the $\mathbf{E}()$ matrix on the left and right boundary as well as in the interior of the space domain. The vector $\mathbf{E}()$ is reserved in the Athena environment to represent the values of these elements. The variable **LEFT** is reserved in Athena to indicate the left boundary location, the variable **RIGHT** is reserved to indicate the right boundary location and the variable **INTERIOR** is reserved to indicate the interior of the space domain. To enter the heading for the Coefficient Matrix Section:

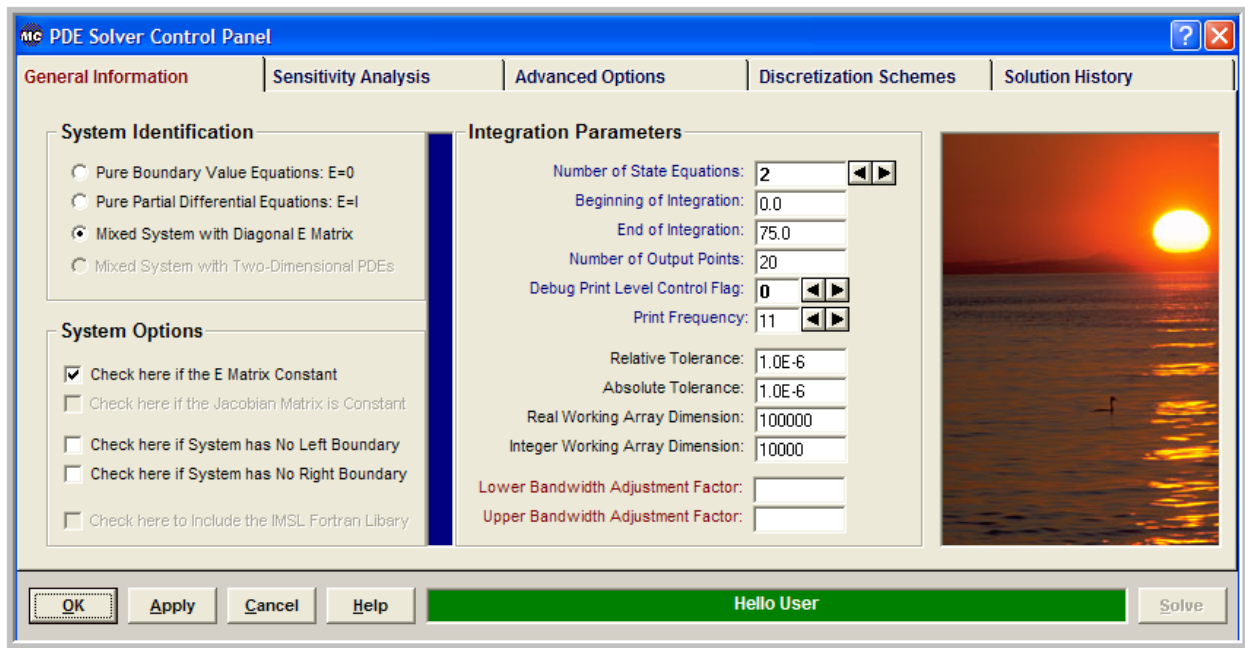
- ❖ From the *Model* menu choose *Coefficient Matrix* (or **Hit F11**)
- ❖ Enter the source code as shown below for our example.

```
@Coefficient Matrix
E(2) = 1.0
If (INTERIOR) Then
  E(1) = 1.0
EndIf
```

The PDAPLUS Solver

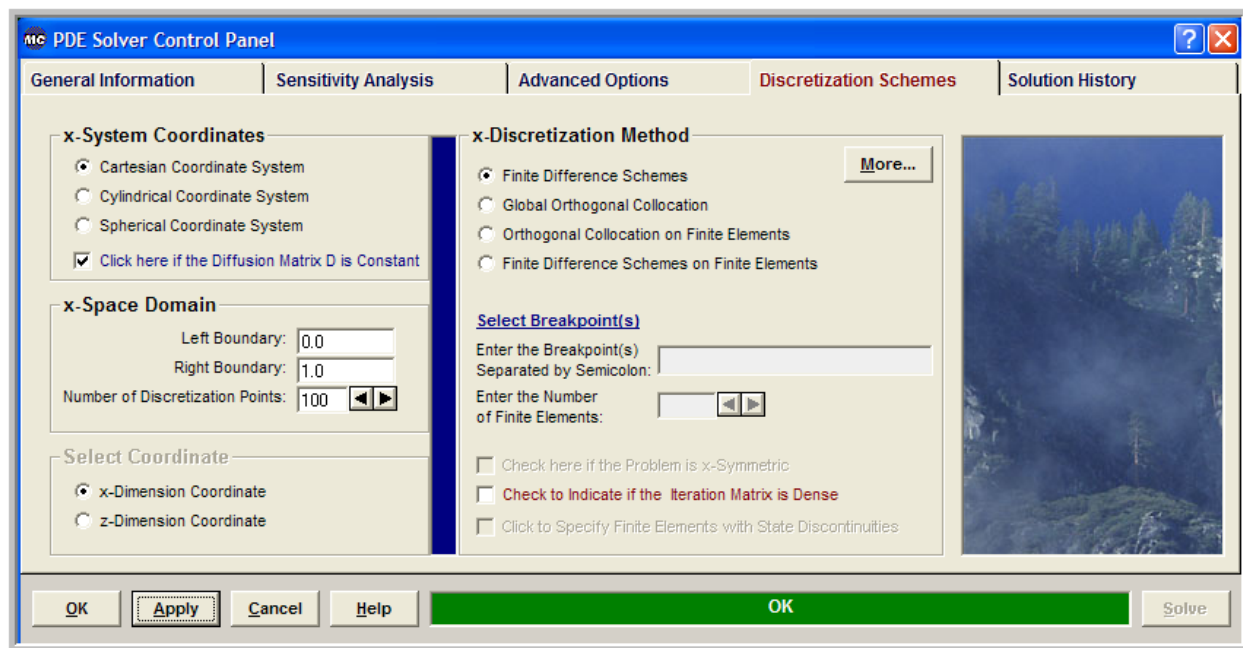
It is now time to access the Athena Visual Studio solver for Partial Differential Equations in order to enter information about the system of equations we wish to solve and various other parameters that control the integration algorithm, To do that:

- ❖ From the *Model* menu choose *Load Solver*, or **Hit F12**
- ❖ The **PDE Solver Control Panel** window appears
- ❖ Enter the solver parameters as shown below for our example



In the **System Identification** group you will see that the option *Mixed System with Diagonal E Matrix* has already been selected for you. From the **Integration Parameters** group enter the Number of State Equations, the Beginning of Integration, the End of Integration and the Number of Output Points (control the granularity of graphs). You may also change the Debug Print Level Control Flag, the Print Frequency (determines how many discretization points are printed) and the Relative and Absolute State Tolerance fields. The Real and Integer Working Array Dimension fields are indicative of the size of the problem. If the default values are not large enough the solver will return with the message indicating the space requirements for your problem. You may also need to examine if your model does not have a left or right boundary, in which case you will have to check the appropriate options in the **System Options** group. After you make all your selections click **OK**.

Next click on the **Discretization Schemes** tab. The following display appears



In the **x-System Coordinates** group select the Cartesian Coordinate System (or whatever is proper for your model) and check the [Click here if the Diffusion Matrix D is Constant](#) to indicate that for our example, the Peclet numbers for heat and mass transfer do not change along the reactor length. From the **x-Discretization Method** group select the *Finite Difference Schemes* method and optionally click on the command button [More...](#) to select *Central Differences*, *Upwind* or *Downwind Differences* (that might be appropriate for hyperbolic partial differential equations); you may also change the *Non-Uniform Grid Attenuation* factor that controls the uniformity of the discretization points distribution along the spatial direction). Finally, from the **x-Space Domain** group enter the *Left Boundary* value, the *Right Boundary* value and the *Number of Discretization Points*. After you make all your selections click **OK**.

Saving and Running


You are now ready to save your model and run it. New files are labeled **UNTITLED** until they are saved. Keep in mind that the maximum number of characters in a line is 132; the maximum number of lines in a file is infinity. In order to save your project:

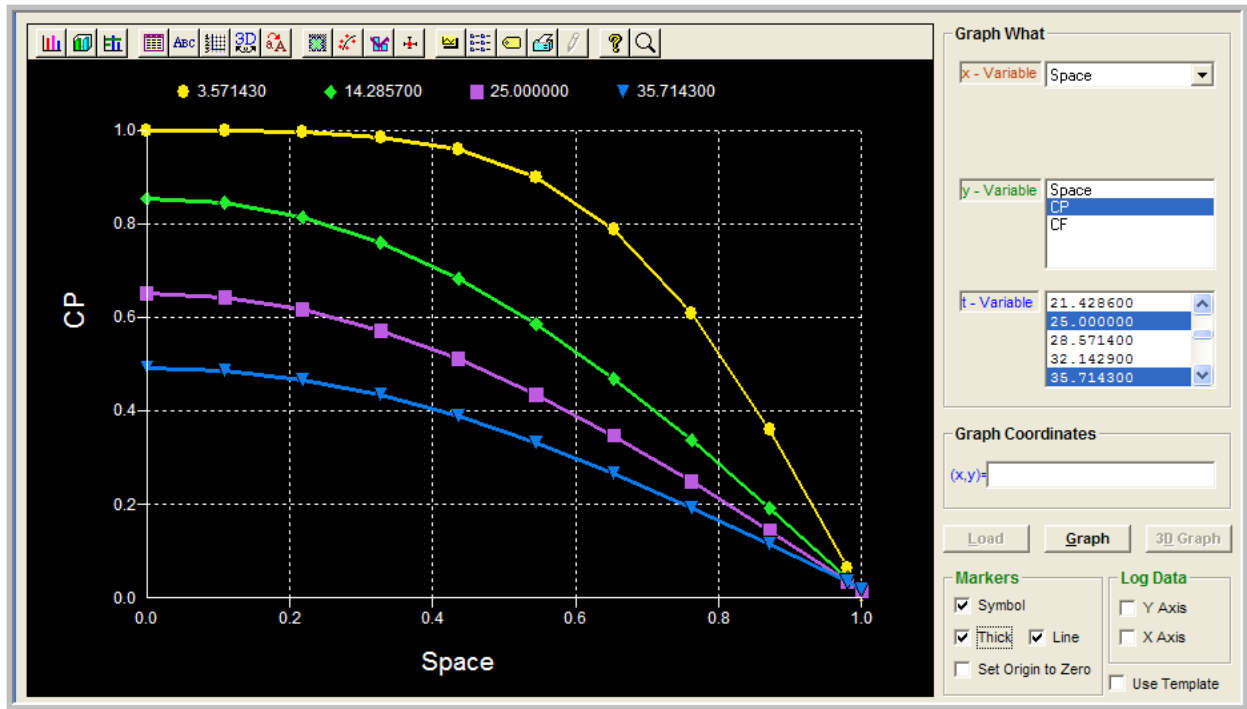
- ❖ From the **File** menu, choose **Save**. The *Save As* dialog box appears. This action saves your model and creates the Fortran code.
- ❖ In the Directories box, double-click a directory where you want to store the source file of your project.
- ❖ Type a filename (a filename cannot contain the following characters: \ / : * ? “ < > |) in the File Name box, then choose **OK**. The default extension is **avw**
- ❖ To view the Fortran code that you created from the **View** menu choose **Fortran Code**.

You may now choose to compile, build and execute your project; to do that:

- ❖ From the **Build** menu choose **Compile** (or **Hit F2**)
- ❖ From the **Build** menu choose **Build EXE** (or **Hit F4**)
- ❖ From the **Build** menu choose **Execute** (or **Hit F5**)

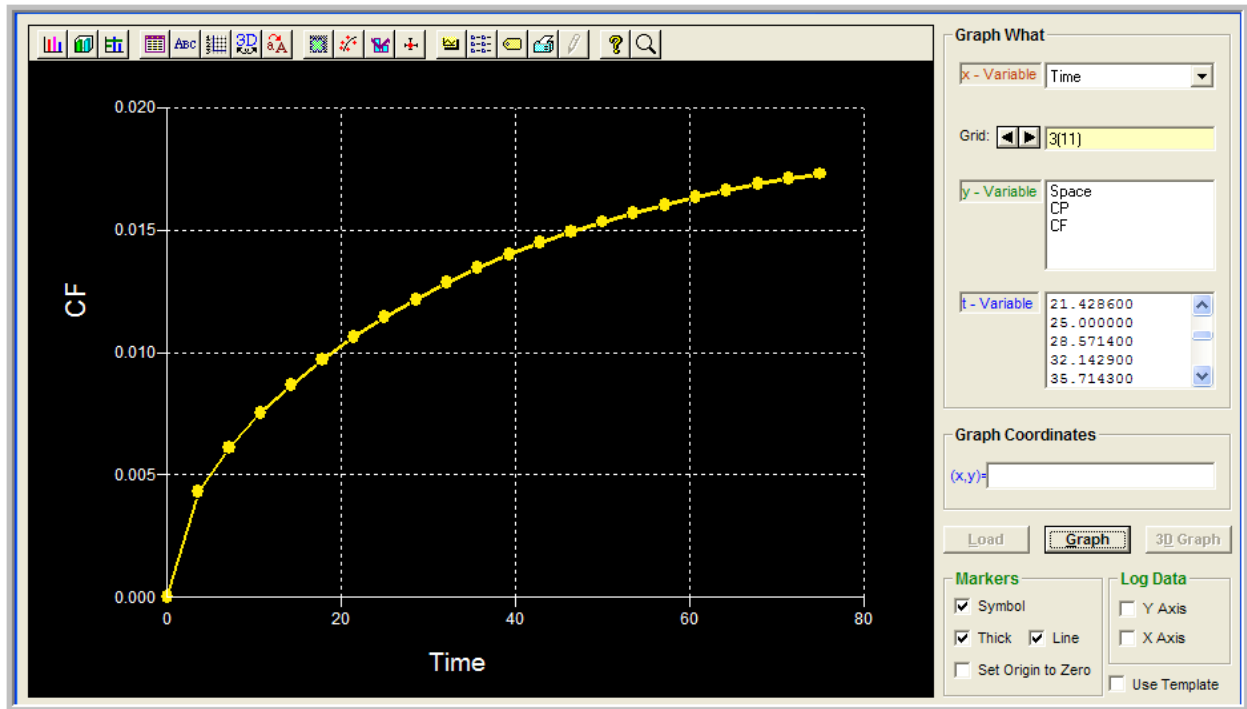
Numerical and Graphical Results

If everything goes well the results window will appear. In this window you can see the solution of your problem as well as various statistics pertaining to the solution process. If you wish to see the time or space profiles for the concentration in the polymer and solution phases, from the **View** menu choose **Solution Graphs**, or click . The graphics control panel appears:



In this window first we click **Load** to load the numerical results. Then in the **Graph What** group we select the **x-variable** (here **Space**) and the **y-variable** (here the polymer concentration **CP**) and in the **t-variable** select a number of times at which you wish to plot the concentration; then click **Graph**. You should see the graph that appears above. You may now click on the graph toolbar and modify the type, title, symbol, the style and miscellaneous other properties of the graph.

Suppose you wish to see the concentration of the migrant in the solution as a function of time. To do that in the **x-variable** select **Time** from the drop down list.



In the **y-variable** double-click on the **CF** (migrant concentration in solution) variable and then press the **Grid** spin control until you reach the discretization point at the right boundary (in this case point number 12). Notice that this discretization point is not equal to 102 because of the selected *Printing Frequency* input number. Then click **Graph**; you should see the graph above that shows the dimensionless concentration of the migrant as a function of time.

Explicit Auxiliary Variables

Auxiliary variables can be implemented in an explicit way, without having to increase the dimensionality of the original system of equations. In order to do that we have to make use of two Athena Visual Studio options; the first allows us to enter custom code right after a call to the integrator, and the second allows us to append auxiliary variables to the state vector for printing purposes.

As an example, consider the introduction of the auxiliary variable *Flux* that represents the flux of the migrant at the particle surface; this auxiliary variable is therefore defined by the following equation:

$$Flux = -D_s \frac{\partial u_1}{\partial x} = -D_s \frac{u_{1,Ngrid} - u_{1,Ngrid-1}}{x_{Ngrid} - x_{Ngrid-1}}$$

The code below illustrates how to do the implementation:

```
@Initial Conditions
U(1)=1.0                ! Dimensionless concentration in polymer
U(2)=0.0                ! Dimensionless initial concentration in solvent

@Model Equations
F(1) = Dv * Uxx(1)
F(2) = 0.0

@Boundary Conditions
If (LEFT) Then
  F(1) = Ux(1)
  F(2) = 0.0
Else
  F(1) = U(1) - Kpf * U(2)
  F(2) = -Ds * Ux(1)
End If

@Coefficient Matrix
E(2) = 1.0
If (INTERIOR) Then
  E(1) = 1.0
EndIf

@After Calling Solver
Global Flux(Mgrid) As Real
Flux=-Ds*(U(1,Ngrid)-U(1,Ngrid-1))/(xGrid(Ngrid)-xGrid(Ngrid-1))

@Solver Options
Tend=75.0
Npts=1
Append=Flux
```

Notice above in the Solver Options section, the **Tend** keyword specifies the end of the integration of the system of differential equations while the **Npts** keyword specifies the number of output points for printing purposes. The **Append** keyword instructs the solver to append the auxiliary variable to the state vector so that they can be printed and therefore used in the graphics server. To view all Solver Options form the **View** menu in Athena select **Solver Options**.

Now from the **Build** menu select **Execute** (or **Hit F5**). You should see the following results:

```

Number of State Equations..... 2
Number of Sensitivity Parameters..... 0
Number of Discretization Points..... 102
Number of Integration Output Points..... 3

```

TIME	SPACE	U(1)	U(2)	Flux
0.00000E+00	0.00000E+00	1.00000E+00	0.00000E+00	2.18160E-02
	1.08911E-01	1.00000E+00	0.00000E+00	2.18160E-02
	2.17822E-01	1.00000E+00	0.00000E+00	2.18160E-02
	3.26733E-01	1.00000E+00	0.00000E+00	2.18160E-02
	4.35644E-01	1.00000E+00	0.00000E+00	2.18160E-02
	5.44554E-01	1.00000E+00	0.00000E+00	2.18160E-02
	6.53465E-01	1.00000E+00	0.00000E+00	2.18160E-02
	7.62376E-01	1.00000E+00	0.00000E+00	2.18160E-02
	8.71287E-01	1.00000E+00	0.00000E+00	2.18160E-02
	9.80198E-01	1.00000E+00	0.00000E+00	2.18160E-02
	1.00000E+00	0.00000E+00	0.00000E+00	2.18160E-02
3.75000E+01	0.00000E+00	4.70675E-01	0.00000E+00	1.53043E-04
	1.08911E-01	4.64598E-01	0.00000E+00	1.53043E-04
	2.17822E-01	4.45338E-01	0.00000E+00	1.53043E-04
	3.26733E-01	4.13471E-01	0.00000E+00	1.53043E-04
	4.35644E-01	3.69953E-01	0.00000E+00	1.53043E-04
	5.44554E-01	3.16090E-01	0.00000E+00	1.53043E-04
	6.53465E-01	2.53502E-01	0.00000E+00	1.53043E-04
	7.62376E-01	1.84075E-01	0.00000E+00	1.53043E-04
	8.71287E-01	1.09901E-01	0.00000E+00	1.53043E-04
	9.80198E-01	3.32213E-02	0.00000E+00	1.53043E-04
	1.00000E+00	1.91909E-02	1.37078E-02	1.53043E-04
7.50000E+01	0.00000E+00	1.85093E-01	0.00000E+00	5.45159E-05
	1.08911E-01	1.82926E-01	0.00000E+00	5.45159E-05
	2.17822E-01	1.76057E-01	0.00000E+00	5.45159E-05
	3.26733E-01	1.64694E-01	0.00000E+00	5.45159E-05
	4.35644E-01	1.49179E-01	0.00000E+00	5.45159E-05
	5.44554E-01	1.29980E-01	0.00000E+00	5.45159E-05
	6.53465E-01	1.07676E-01	0.00000E+00	5.45159E-05
	7.62376E-01	8.29386E-02	0.00000E+00	5.45159E-05
	8.71287E-01	5.65142E-02	0.00000E+00	5.45159E-05
	9.80198E-01	2.91992E-02	0.00000E+00	5.45159E-05
	1.00000E+00	2.42014E-02	1.72867E-02	5.45159E-05

EXIT PDAPLUS: SOLUTION FOUND

```

Number of Steps Taken Thus Far..... 246
Number of Function Evaluations..... 736
Number of Jacobian Evaluations..... 32
Number of Jacobian Factorizations..... 32

```

From the above table we clearly see how the Athena Visual Studio solver augmented the auxiliary variable to the state vector for printing purposes. One can now access the Graphics control panel to graph the auxiliary variables along with the state variables.